

IMF STAFF DISCUSSION NOTE

# When Should Public Debt Be Reduced?

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## EXECUTIVE SUMMARY

Financial bailouts, stimulus spending, and lower revenues during the Great Recession have resulted in some of the highest public debt ratios seen in advanced economies in the past 40 years. Recent debates have centered on the pace at which to pay down this debt, with few questions being asked about the desirable level of public debt to which the economy should converge following a debt shock. While some countries face debt sustainability constraints that leave them little choice, others are in the more comfortable position of being able to fund themselves at reasonable—even exceptionally low—interest rates. For these countries, there is a very real question of whether to live with high debt while allowing the debt ratio to decline organically through growth, or to pay it down deliberately to reduce the burden of the debt.

This paper considers optimal public debt and investment policy in the aftermath of the global financial crisis. It abstracts from rollover risks faced by countries that are near their debt limits, and also from shorter-run cyclical considerations. It is not that these considerations are unimportant—for they surely are at present in a number of countries. But they are not the dominant factors for countries that are near full employment and enjoy considerable fiscal space (even in some cases despite relatively high levels of public debt, as argued in our earlier IMF Staff Position Note on this topic, Ostry and others [2010]), a set of countries that are also in need of policy advice.

Under these conditions, economic theory provides three insights. First, inherited public debt, though accumulated for good reasons, represents a deadweight burden on the economy, dimming both its investment and growth prospects; a corollary is that an economy that has inherited a lot of public debt (for example, because of a financial crisis) will rationally choose to invest less in public capital than one with a lower level of debt. Second, if fiscal space remains ample, policies to deliberately pay down debt are normatively undesirable. The reason is that for such countries, the distortive cost of policies to deliberately pay down the debt is likely to exceed the crisis-insurance benefit from lower debt. In such cases, debt-to-GDP ratios should be reduced organically through growth, or opportunistically when less distortionary sources of revenue are available. Third, public debt should be issued to smooth the taxes necessary to finance lumpy expenditures. This action yields a version of the golden rule whereby public investment is debt-financed and undertaken to the point that social returns equal the market interest rate, with the twist that the social return will itself be reduced by the need to raise distortive taxation on labor and capital to service the higher debt.

What constitutes a safe level of debt (or ample fiscal space, as defined in Ostry and others [2010]) is, needless to say, very difficult to pin down precisely in practice, and can never be established through some mechanical rule or threshold. Stress testing public-sector balance sheets is essential to form judgments at the country level of what constitutes a safe public debt level. It may be helpful to think of debt levels as falling into three zones: a green zone, in which fiscal space is ample; a yellow zone, in which space is positive but sovereign risks are salient; and a red zone, in which fiscal space has run out. This paper is concerned with green-zone cases. Reducing debt in such cases is likely to be normatively undesirable as the costs involved will be larger than the resulting benefits.

## INTRODUCTION

High public debt in advanced economies is an important legacy of the global financial crisis, as is an erosion of the public capital stock (although this latter problem started a couple decades ago, and applies to not only several advanced economies, but also many emerging market economies and low-income countries, in which infrastructure bottlenecks and deficiencies are a hindrance to economic development). The first problem points in the direction of reducing public debt, while the second (given that economic theory strongly suggests that investment should be financed by debt rather than taxes) points to increasing it. This latter issue is especially true at the current juncture, given the very low level of real interest rates and the existence of demand shortfalls in a number of advanced economies.

Why do we think that high public debt should be reduced? The main rationale is essentially one of risk management, the desire for additional margins to cope with unanticipated or contingent risks. The option value of lower debt is particularly high if there are risks of catastrophic events (an example would be a financial crisis in which a public backstop is essential), in which the government would need to ramp up borrowing massively (the more so given the political and economic limits to raising taxes sharply in a pinch). If debt is high when such a shock occurs, a heavy penalty may be exacted as sovereign risk premiums rise and, in extreme cases, a shutout from markets would ensue. In other words, debt needs to be reduced today to lower the potential risk of a sovereign crisis tomorrow.

A second rationale for why high public debt needs to be brought down is the belief that high public debt weighs on economic growth. While causality runs both ways, an important causal channel is taxation: high public debt implies the need to distort economic activity (labor, capital) to service the debt (either through taxation or cuts in productive spending), which dampens economic growth. A reasonable idea is that laying the foundation for sustainable growth requires paying the upfront cost of reducing the debt today.

For countries with a demand shortfall and infrastructure gaps, but with high and risky levels of public debt, there is thus an evident tradeoff between building up the public capital stock and containing sovereign risks and laying the foundation for future growth. An exception to the dilemma would be if debt-financed public investment spending actually lowered the debt ratio, as can occur if Keynesian effects are large, the interest burden is minuscule, and investment is super-efficient (the argument is not fundamentally different from the one put forward by supply-siders in the 1980s, that tax cuts would raise GDP and government revenue so much that the debt ratio would fall). While these conditions may be salient in some countries at present, few consider them to constitute a set of neutral assumptions that should guide public policy in general. More likely, the typical case is one in which the 'lunch' of a public infrastructure buildup has to be paid for with higher debt, and thus a judgment formed about the weight to attach to the different fiscal objectives in specific cases.

This is why debates at present seem focused on what is the greater priority: lowering public debt or building public infrastructure. For countries with low debt and big infrastructure needs (and with idle resources and facing low interest rates), building infrastructure should be the greater priority. For countries with significant risk of fiscal distress, it is unlikely that they could afford to take the chance of going on a borrowing spree, no matter how large the public investment deficiencies. But what about less clear-cut cases? What about countries with high debt, but no plausible risk of fiscal distress, with some infrastructure needs, but that are operating near full employment and normal levels of real interest rates? Perhaps the largest economies are not in this situation today, but these conditions are not far-fetched, and have prevailed for many countries in the past. What policy advice applies to such countries?

To get at this issue, one needs an analytical framework that abstracts from the considerations that have tended to drive recent debates: Keynesian demand management and risk of fiscal crisis (conceptually, these can be added back into the model, but we want to focus on a separate set of issues, which have not been explored much to date). Our approach allows one to focus on how public debt, public investment and growth interact in situations in which the economy may be near full employment and there is ample fiscal space (which can occur, as argued in our earlier IMF Staff Position Note, Ostry and others (2010), even if there is substantial public debt—Figure 1).<sup>2</sup> Two questions seem especially relevant in such a framework.

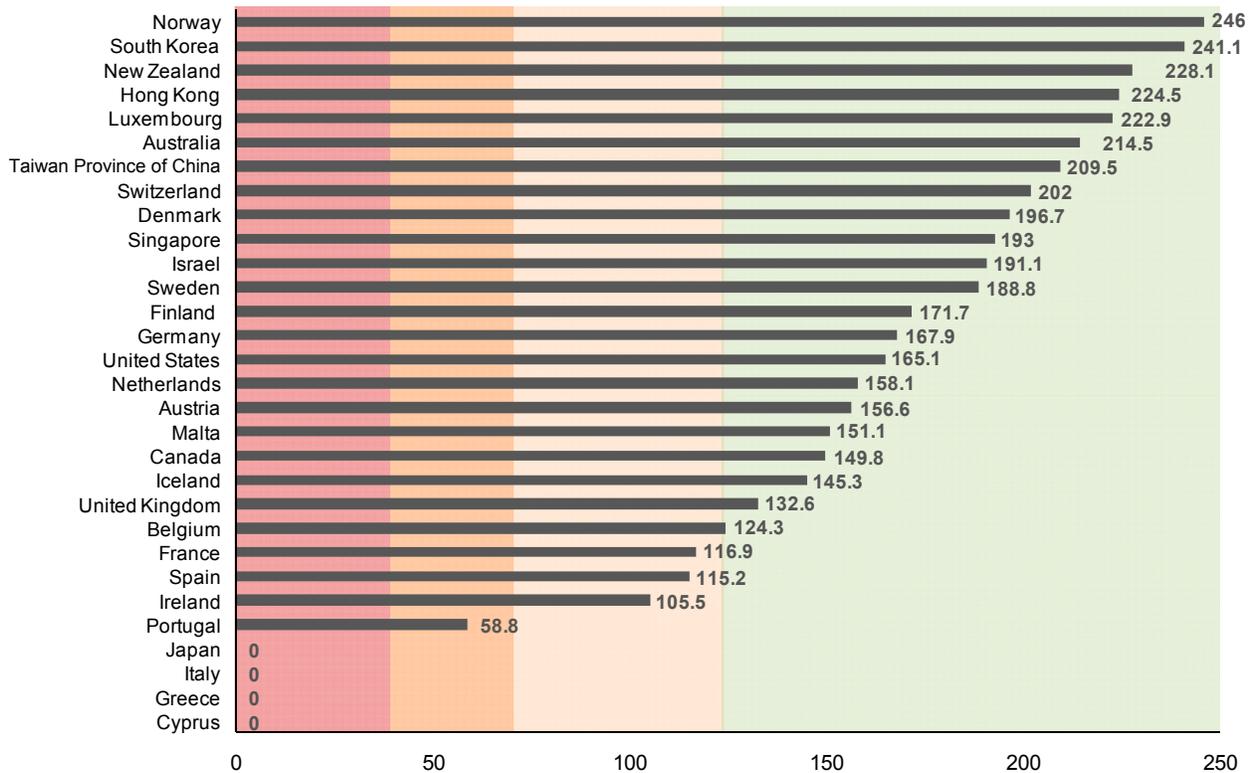
The first relates to the normative implications of high public debt itself. One implication might be that it should be paid down in a reasonable timeframe (not too fast so as to open up an output gap, for example), not only because of material effects of sovereign risk, but also because the debt itself represents a burden in the classic public finance sense—distortive taxation is needed to service it, undercutting the foundation of future economic growth. A fiscal effort today saves on the fiscal burden in perpetuity, providing a benefit equal to the present value of the smaller distortion. Against this argument is the increase in distortionary taxation today (or during the timeframe of the fiscal effort), which is costly. Does a standard public-finance model yield a prediction of the relative size of the gain and losses? And how does this prediction relate to how to handle the debt shock?

A second issue concerns the impact of the debt shock on the warranted level of public investment. Higher debt, as just mentioned, requires higher distortive taxation for servicing. Such taxation is likely to reduce the productivity of labor and capital (factors that are complementary to public capital), meaning that both output and public capital should be lower than in a situation in which there is less public debt. Thus, there are implications of public debt for warranted public investment and, in turn, for the growth path of the economy.

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<sup>2</sup> The discussion of fiscal space in Ostry and others (2010) has since been adopted by Moody's in their assessment of fiscal risks in advanced economies. Both analyses highlight the existence of wide gaps (margins of maneuver) between current debt ratios and public debt limits (points at which market access is likely to be curtailed or sharp rises in sovereign borrowing costs take place) in a number of advanced and emerging market economies.

**Figure 1. Fiscal Space: Distance to Debt Limit**  
(percentage points)



Source: Moody's Analytics (estimates as of May 2014, <https://www.economy.com/dismal/fiscal-space/default.aspx>) based on methodology developed in Ostry and others (2010) and Ghosh and others (2013).

Note: Fiscal space is defined as the difference between the debt limit and the actual debt-to-GDP ratio.

Our benchmark setup is a closed-economy representative-agent model of domestic debt, so we abstract from inter- and intragenerational distribution issues, as well as the international transfer problem.<sup>3</sup> The only purpose of public debt is to shift intertemporally the costs associated with distortionary. Despite the abstraction of the formal framework—or perhaps because of it—we are able to draw several clear policy-relevant insights.

<sup>3</sup> In an open economy, paying down external debt involves a costly transfer of resources abroad, so the closed-economy assumption actually stacks the argument against “living with the debt.” Note that, since the analysis here pertains solely to sovereigns that are firmly in the “green zone” of ample fiscal space, crisis risk is ruled out regardless of whether the economy is open or closed: inasmuch as the government may be less willing to make an extraordinary fiscal effort to honor its debt if held by foreigners rather than domestic residents, the size of the green zone could conceivably be narrower in the open economy context. As to tax incidence, in an open economy in which capital is mobile, it would be very difficult to tax that capital, but this is economically very similar to the closed-economy case in which the optimal capital tax rate falls to zero after the initial period (as in Chamley [1986]). Essentially, in the closed economy, capital is intertemporally mobile (investment will not materialize if capital is taxed), while in the open economy, it is mobile across borders, with the same result (it is very difficult to tax). Thus, we do not see sharply different messages as far as tax incidence is concerned.

- **Inherited public debt represents a deadweight burden on the economy, reducing both investment potential and growth prospects.** Although the debt may have been incurred for good reasons, for a given stock of public capital, the higher the inherited debt, the poorer the economy (by the present value of the distortionary costs of the taxation needed to service the debt). Efficiency dictates that the larger the inherited debt, and thus the higher the level of taxation, the lower will be both public and private investment, and the lower will be output growth. Higher-debt economies will rationally invest less in public infrastructure than less-indebted economies.
- **Where countries retain ample fiscal space, governments should not pursue policies aimed at paying down the debt, instead allowing the debt ratio to decline through growth and “opportunistic” revenues, living with the debt otherwise.** The reason is that the deadweight loss associated with inherited public debt represents a sunk cost—so, abstracting from rollover risk, there is little purpose in paying it down by raising taxes or cutting productive government spending (of course if there is scope to cut unproductive spending this should be pursued). Distorting your economy to deliberately pay down the debt only adds to the burden of the debt, rather than reducing it. When fiscal space is limited, incurring this cost is likely to be normatively desirable given the crisis-insurance benefit. When space is ample—which cannot be established through some mechanical rule but will generally require judgments based on stress testing fiscal balance sheets to withstand extreme shocks—the distortive cost of paying down the debt is likely to exceed the crisis-insurance benefit.
- **Debt should be used to smooth the taxes necessary to finance lumpy government expenditures.** For public investment, this implies debt-financing of projects whose social marginal product earns at least the market interest rate. The twist on the standard “golden rule” is recognizing that the additional debt incurred will need to be serviced with distortionary taxation of factors (labor, capital) that may be complementary to public capital, thereby reducing the return to the public investment. Thus, the social rate of return will need to be higher as the public debt increases: while the quantitative impact on the required rate of return is likely to be small (in the order of a few basis points), the cumulative effect on the long-run stock of public (and private) capital will not be negligible. For plausible parameters, an increase in public debt of 50 percent of GDP (roughly the order of magnitude experienced by many advanced economies since 2007) would lower steady-state output and consumption by 2 percentage points in perpetuity (implying a present value welfare loss of 30 percent of initial consumption), with a similar decline in long-run levels of private and public capital.

It is a feature of the framework adopted here that higher public debt leads to lower investment, slower transitional growth, and a lower long-run level of output: debt is bad for growth. Despite the clear causality, it does not follow that once the debt has been accumulated, it should be paid down to restore growth. On the contrary, where countries retain ample fiscal space, the cure

would seem to be worse than the disease—the taxation needed to pay down the debt will be more harmful to growth than living with the debt, and the reduction in sovereign risk that would ensue is likely to be smaller than the distortive cost involved in paying down the debt.

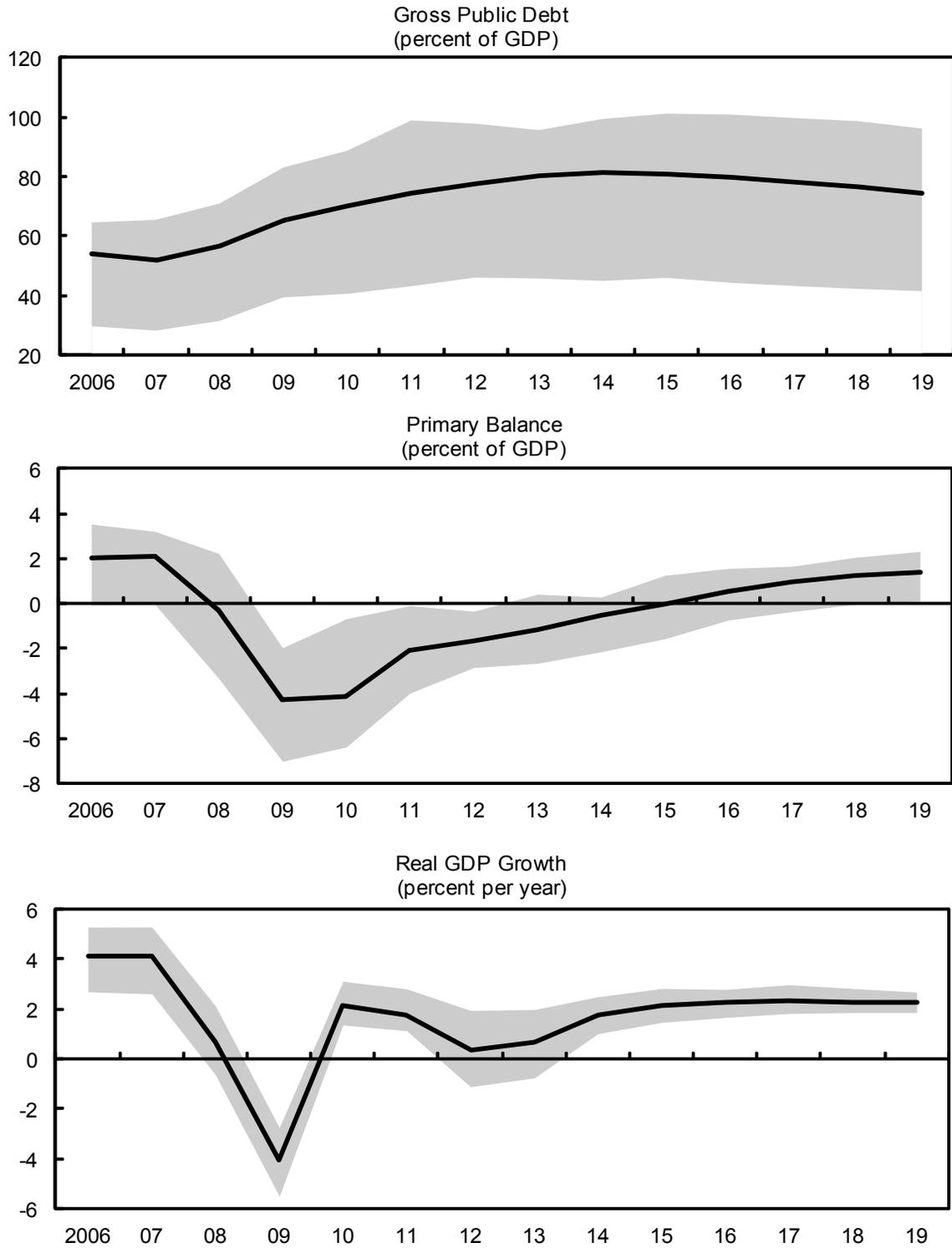
## **PUBLIC FINANCES AND GROWTH IN ADVANCED ECONOMIES: A SNAPSHOT**

The global financial crisis has resulted in sharp increases in advanced-economy public debt ratios, on a scale unprecedented in peace time. On average, debt rose from 53 percent of GDP at end-2007 to almost 80 percent by end-2012, while for the top quartile, debt now exceeds 100 percent of GDP (Figure 2, top panel). Corresponding to this increase were below-the-line operations (mainly financial-sector restructuring costs) together with marked deteriorations in primary balances, which swung from an average surplus of 2.1 percent of GDP in 2008 to a deficit of 4.4 percent in 2009, before recovering partially by 2012 (Figure 2, middle panel). In turn, the deterioration in primary balances corresponded mainly to the loss of revenues and the operation of automatic stabilizers during the Great Recession; very little represented discretionary stimulus, and of that, only a small fraction was investment in public infrastructure. Thus, while the accumulation of public debt was generally for good reasons (averting an economic or banking system collapse), the fact remains that most advanced economies have built up large stocks of debt but have little or no more public infrastructure to show for it.

Along with the deterioration of the public finances came the collapse of output growth. Real GDP growth in advanced economies turned sharply negative in 2009, rebounded somewhat in 2010, and has remained moderate thereafter. Across countries, many different factors were at play in the initial growth collapse, which certainly cannot be attributed to the rise in public debt (the causality was almost surely the other way around). But longer term, it becomes more plausible that the rise in public debt may have effects on investment and growth. It is noteworthy, for instance, that projections for 2017—a full decade after the onset of the crisis—suggest that advanced economies will have barely half their precrisis growth rates, and projected growth for 2013–17 is strongly negatively correlated with end-2012 public debt ratios (the correlation is  $-0.41$ , and statistically significant at the 5 percent level).

These developments raise important policy questions: will the rise in public debt dim investment and growth prospects? If so, should governments seek to pay down their debt as soon as possible? Or should they embark upon ambitious public infrastructure programs to help restore growth, taking advantage of low interest rates? Answering such questions requires a formal framework based on public finance theory, to which we now turn.

Figure 1. Advanced Economies: Fiscal Developments and Growth <sup>1/</sup>



Sources: IMF, World Economic Outlook and Fiscal Monitor databases.  
 1/ Average, and upper and lower quartiles.

## A PURE THEORY OF PUBLIC DEBT AND INVESTMENT

To analyze the normative aspects of public debt it is useful to abstract from inter- and intragenerational distribution issues as well as the international transfer problem, and focus on the situation where the debt is one that we owe ourselves.<sup>4</sup> To yield policy insights, the framework must incorporate two features: at least part of government spending should be on productive public capital; and only distortionary taxes should be available to finance spending. In such a setup, public borrowing does not relax the economy's flow resource constraint, and public spending necessarily crowds out private spending. The purpose of public debt is to shift the burden of taxation over time so as to reduce its total distortive cost.

### Optimal Fiscal Policy

A formal model with these features is presented in Box 1. The representative agent chooses how much to consume and to work, as well as investment in physical capital and government bonds. The first-order conditions characterizing behavior are standard, and imply that labor supply will depend on the after-tax wage; the demand for government bonds will be a function of the interest rate; and private investment will depend on the after-tax return on capital.

The government must decide how much to invest in public capital, how much of the public consumption good to provide, what taxes to levy on wages and on the return to private capital, and residually, how much to borrow (Box 2). Public debt is in the form of one-period bonds and there is no possibility of default. The government is benevolent in the sense that it chooses its fiscal policy to maximize the representative agent's lifetime utility, subject to the economywide resource constraint, its own budget constraint, and a "feasibility constraint." The latter summarizes the agent's endogenous response to taxation and its intertemporal budget constraint. For instance, if the government taxes wages more heavily, labor supply will be reduced, and this needs to be taken into account in the government's optimization problem. If non-distorting taxes were available, the feasibility constraint would never bind.

The first-order conditions characterizing optimal fiscal policy are analogous to those of the representative agent, except that private marginal utilities and rates of substitution get replaced by their corresponding social quantities. The latter differ from the private counterparts because of the feasibility constraint (that is, because only distortionary taxes are available). Thus the public consumption good should be provided to the point that its marginal utility equals the *social* marginal cost of its provision, and public investment should be undertaken to the point

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<sup>4</sup> In practice, of course, it makes a great deal of difference who is the "we" and who is the "ourselves." The assumption that debt is held by domestic residents is plausible for advanced economies where, on average, more than two-thirds of public debt is held by residents (ranging from about 30 percent in France and Germany to 70 percent in the United Kingdom and the United States, and more than 90 percent in the advanced economy with the highest debt ratio, Japan). On the transfer problem, see Keynes (1929).

that the marginal product of public capital equals the *social* marginal rate of substitution across periods. Taxes on labor and capital should be chosen such that the marginal disutility of work is equal to the marginal *social* value of producing more output, and the marginal product of private capital is equal to the *social* marginal rate of substitution across periods. Although social marginal utilities and rates of substitution differ from the private quantities because taxes are distortionary, a key result that emerges from the formal optimization problem is that the government's implied discount rate will equal the market interest rate. (In essence, the wedge between private and social valuations is constant over time, and therefore does not affect the discount rate.<sup>5</sup>) As discussed next, this has important implications for optimal debt policy.

An illustrative baseline scenario is depicted in Figure 3 (solid line) where, for ease of exposition, it is convenient to start with a case where there is no taxation of private capital. Starting from a low level of public capital, the government invests, raising output and consumption of both the private and public goods until the steady state is reached.<sup>6</sup> The economy thus follows a standard neoclassical growth pattern. (It would be straightforward to add exogenous technical change, in which case the economy would be constantly converging on a shifting rather than a unique steady state, but this is inessential to the analysis here.)

Turning to public debt dynamics, there is a sharp initial increase in debt (especially when the starting level of debt is high, dashed lines), followed by more steady increases along the optimal path. Since the distortionary costs of taxation are convex (that is, increasing at an increasing rate) in the tax rate, minimizing the total distortionary cost involves smoothing tax rates over time. The reason debt is issued here is thus two-fold: first, public investment tapers off over time, implying a declining path of investment expenditure; second, growth of the economy implies a larger tax base (wage income). Both effects mean that it is optimal to defer payment for government expenditure by issuing public debt. Indeed, it can be shown that a "golden rule" obtains whereby there is an approximate one-to-one correspondence between public investment and cumulative primary deficits. Thus public investment is largely debt financed, and undertaken to the point that its marginal product is equal to the market interest rate on public debt. This is the standard "golden rule" for public investment, although it bears emphasizing that the debt will need to be serviced using distortionary taxation on factors complementary to public capital (for example, labor, private capital), thus reducing the marginal product of public capital. While the quantitative impact on the required social rate of return is likely to be small, the cumulative effect on the stock of public capital is non-negligible at higher values of inherited debt.

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<sup>5</sup> This result, originally due to Chamley (1986) for the long-run and to Chari, Christiano, and Kehoe (1994) in the short run for a CES (constant elasticity of substitution) utility function, is akin to Little and Mirrlees' (1974) argument that, even in countries with highly distorted capital markets, world interest rates can be used for discounting returns in project evaluation.

<sup>6</sup> Output declines slightly as the steady state is approached because leisure is a normal good, so the representative agent works less as the economy becomes richer.

## A Debt Shock

With these preliminaries, we can turn to our question of interest: what happens if there is an exogenous increase in public debt? (The increase is exogenous in the sense that it does not correspond to higher public investment or provision of the public good, but rather results from some extraneous event, such as the fallout from a financial crisis.) Conceptually, there are three possibilities. Optimal policy could involve paying down the debt immediately so that, following the initial period, the debt path returns to the baseline scenario; or it could involve paying down debt more gradually, so there is gradual convergence to the baseline scenario; or the economy could simply live with the higher debt forever. A priori, it is far from clear which will obtain. On the one hand, paying down the debt quickly involves high rates of taxation and correspondingly large distortionary costs; on the other hand, never paying it down means servicing it forever, thus incurring the distortionary costs of taxation in perpetuity. To see which is optimal, the model outlined in Box 1 is solved numerically, and the effect of an initial debt shock simulated.

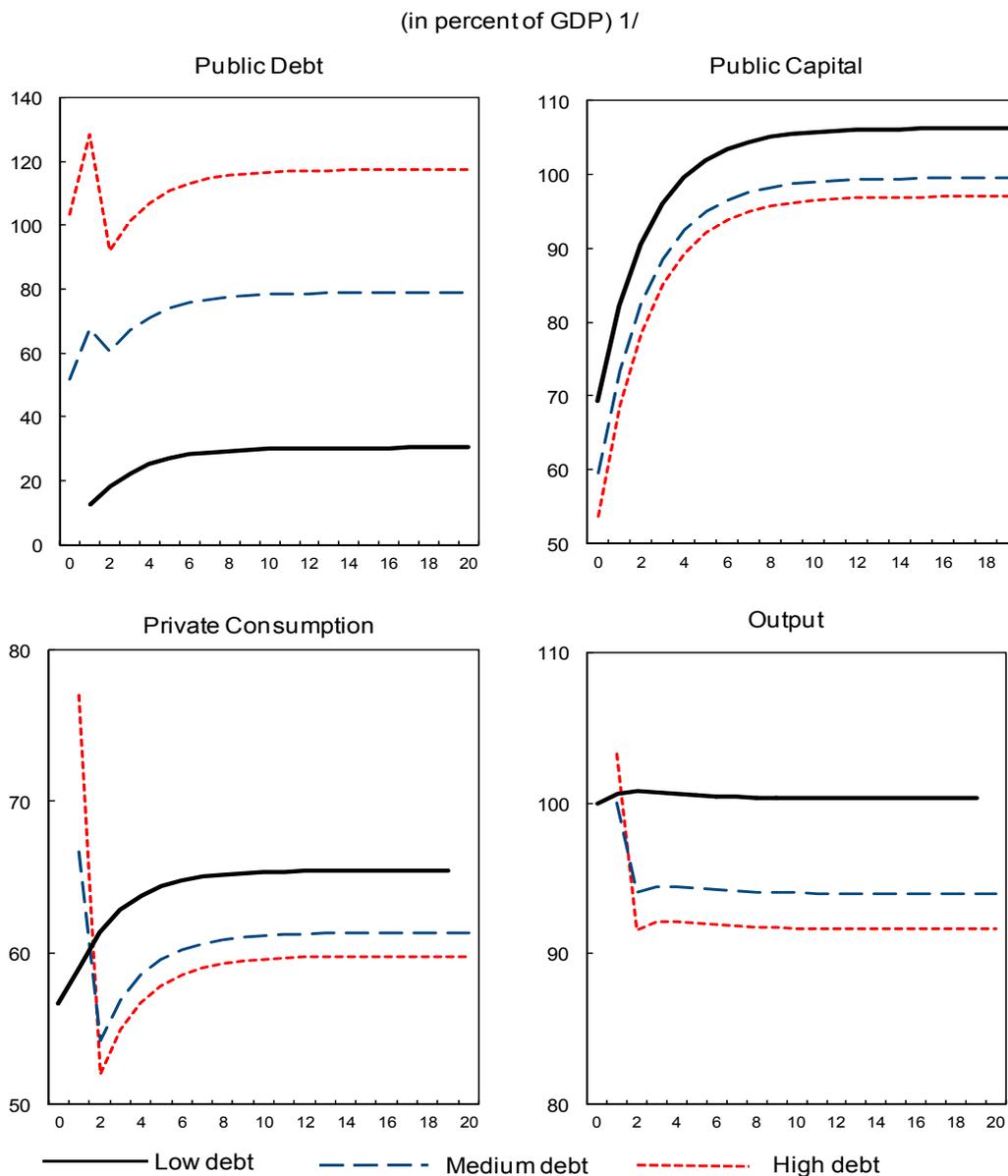
It turns out that the optimal policy involves living with the debt forever, so the new path of debt is essentially “parallel” to the baseline path (Figure 3). To understand why, recall the key result from the optimization problem that despite private and social valuations being different, the government’s discount rate will equal the market interest rate. Suppose, in any period, the government is considering paying down \$1 of public debt. If it does so today, it incurs the distortionary costs of raising another dollar of revenue. If it defers repayment to tomorrow, the debt will have grown by the market interest rate,  $(1+r)$ , and the cost will be the distortion associated raising an additional  $\$(1+r)$ . But the government discounts the future at precisely the market interest rate, so it gains nothing by paying down a dollar of debt today.<sup>7</sup> Since the same argument can be applied to any period, the government will just live with the original inherited debt. In fact, applying Robert Barro’s (1979) tax-smoothing argument, it can be shown that taxes will be set so as to achieve a constant distortionary cost; once the steady state is achieved, this will imply constant tax rates and revenues, the latter equal to the interest payments on the debt (if taxes were set any higher, debt would decline, so eventually taxes would decline, violating the tax-smoothing principle; if set any lower, debt would grow explosively, eventually requiring higher taxes, which again would violate the tax-smoothing principle).

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<sup>7</sup> In this simple heuristic explanation, the interest rate is treated as constant; in the actual optimization in Box 2, the government takes into account the endogeneity of the interest rate to the private sector’s saving decision and its own fiscal policies. Since there is no possibility of default here, there is no risk premium. In a fuller model, the government would also need to consider that, as debt rises and the debt limit is approached, so will the risk premium; the government would in this fuller setup take this into account in solving for its optimal fiscal program (consumption, investment, borrowing, taxation). The theoretical analysis in Ostry and others (2010) and Ghosh and others (2013) suggests that the risk premium only rises appreciably close to the debt limit, and the empirical analysis in Acharya, Drechsler, and Schnabl (2014) implies only modest increases in the risk premium with higher debt. Since the analysis in this paper is solely for countries with ample fiscal space, these considerations are not incorporated here.

Even though it is optimal for the government to live with the debt, this inherited debt is not without its (negative) consequences. An economy that inherits a stock of public debt is “poorer” by the present value of the distortions associated with raising the revenue to service that debt. Higher initial debt requires higher taxation to service it. Higher tax rates lower labor supply, reducing the marginal product of (private and public) capital, with correspondingly lower investment and output. Since the two economies in the simulations start at the same level of output, average growth in the economy with higher initial debt must be lower than average growth in the low-debt economy. Debt is thus bad for growth here, with the causality running from higher debt leading to lower growth via the higher taxation required to service it.

**Figure 3. Dynamics Under Low, Medium, and High Initial Debt**



1/ All variables expressed in percent of initial GDP of the low-debt scenario

Nor is this effect negligible. To put the magnitudes in perspective, the simulation assumes that initial debt is some 50 percent of GDP higher than in the baseline (that is, 100 percent of GDP rather than 50 percent of GDP). As a result, in the steady state, consumption, output, the capital stock, and investment are about 2 percentage points lower permanently compared to the counterfactual. At reasonable discount rates, *the present value welfare cost* of inheriting 50 percent of GDP higher debt is about *30 percent of the initial period consumption*.

### **Paying Down the Debt**

The analytical framework implies that it is better to live with high debt than to pay it down. In part, however, this is because the model does not incorporate rollover risk. In reality, there is a trade-off: paying down the debt may be costly, but doing so lowers the likelihood of a funding crisis. In undertaking the cost-benefit calculus, a first question is how much *more* costly would it be to pay down the inherited debt than to live with it? For plausible parameters, paying down 5 percent of GDP (that is, one-tenth of the additional inherited debt<sup>8</sup>) in one year incurs a present value welfare cost equal to about 1 percent of GDP.<sup>9</sup> But these costs are highly non-linear: paying down 10 percent of GDP of the debt would imply a welfare cost equal to 2–3 percent of GDP, and paying down 20 percent of GDP would incur a cost of at least 6 percent of GDP.

What about the benefit? This is very dependent on the level of the debt and the distance to the sovereign's debt limit—that is, the available fiscal space (as defined in our earlier IMF Staff Position Note on fiscal space, Ostry and others [2010]). If fiscal space is ample, the benefit in terms of lowering the probability of crisis by reducing debt is likely to be small. Of course, in reality, assessing when fiscal space is ample (as Moody's have done for the countries in the green zone of Figure 1—based on the methodology of Ostry and others [2010]) is not a mechanical exercise. In particular, assessing whether pernicious nonlinearities may begin to take hold requires detailed stress testing of public sector balance sheets and judgments about economic resilience in the face of extreme shocks. It may thus be helpful to think in terms of countries falling broadly into three zones: a green zone in which fiscal space is ample and the potential for a sovereign crisis negligible; a yellow zone in which space is positive and perhaps even sizable but where, especially given the likelihood that markets may give little warning of a debt limit (as shown in Ostry and others [2010] and Ghosh and others [2013]), sovereign risks are salient; and a red zone, in which fiscal space has run out. As debt rises, moreover, countries may transition across zones, and the cost-benefit calculus of whether to pay down the debt or not will evolve,

<sup>8</sup> Paying down less incurs a smaller cost, but of course would contribute to correspondingly smaller crisis-reduction benefits.

<sup>9</sup> Of course, if we were thinking in terms of an open economy and external debt, the welfare cost of paying down 5 percent of GDP in external debt would be this distortionary cost (1 percent of GDP) plus the 5 percent transferred to foreigners (so 6 percent). The cost of paying down the debt would thus be much larger in an open-economy setting than in the one here. In that sense, the closed-economy setting actually stacks the argument against living with the debt (since the cost of repaying the debt is much lower in the closed-economy setting).

particularly as the risk premium and borrowing costs begin to rise through the yellow zone. The model in this paper pertains to countries in the green zone, but the practical application would factor in where countries lie along the fiscal space continuum.

Empirically, it is difficult to pin down the probability of a crisis—let alone how much that probability falls as debt declines—because sovereign debt crises in advanced economies are rare. A recent database (Baldacci and others [2011]) of negative fiscal “events” (sovereign default, debt restructuring, spike in spreads, or high inflation), however, can help. For an advanced economy with 120 percent of GDP of debt, the likelihood of a negative event is estimated to be about 2.6 percent per year: over a 20 year horizon, the expected number of crises is 0.52, with a very generous estimate of the output cost associated with these events (most of which are not a full blown sovereign debt crisis) being approximately 15 percent of GDP.<sup>10</sup> At debt of 120 percent of GDP, therefore, the expected loss is 7.8 percent of GDP. Suppose debt is lowered to 100 percent of GDP. The corresponding likelihood of an event is 2.4 percent per year, yielding an expected loss of 7.2 percent of GDP.<sup>11</sup> In other words, the expected benefit of reducing debt from 120 percent of GDP to 100 percent of GDP is only about 0.4–0.6 percent of GDP—or around *one-tenth* of the welfare cost due to distortionary taxation.<sup>12</sup>

The calculation above is very much predicated on the sovereign being sufficiently far from its debt limit (that is, having ample fiscal space). Obviously, at or very near the debt limit (in the yellow zone, but close to the red zone threshold), there will be enormous gain from reducing debt before a crisis occurs. Thus the conclusion is *not* that it is never worth reducing debt in order to lower the likelihood of crisis. Indeed, with the precise location of the debt limit uncertain, there is insurance value in keeping fiscal space against the possibility of future shocks that increase the sovereign’s debt (this would apply to countries in the yellow zone). But for countries that have ample fiscal space (clearly in the green zone), the benefit of reducing debt

<sup>10</sup> The output cost is calculated as the undiscounted sum over the three years subsequent to the event of the difference between actual and precrisis trend GDP. This estimate is in line with other studies—for example, Sturzenegger (2004); Borensztein and Panizza (2008); Levy-Yeyati and Panizza (2011), although a few papers—for example, Furceri and Zdzienicka (2011); De Paoli, Hoggarth, and Saparta (2009) suggest a permanently lower GDP level and thus larger costs.

<sup>11</sup> An alternative approach to estimating the likelihood of crisis is to use the probability implied by market spreads. Simple regressions of government bond yields on debt (with country-fixed effects and time effects) for a panel of advanced economies suggests that an increase in debt of 10 percent of GDP would be associated with 20 basis point higher spreads, so reducing debt from 120 percent of GDP to 100 percent of GDP would be associated with 0.4 percentage points lower spreads. Acharya, Drechsler, and Schnabl (2014) obtain very similar estimates: in their study, 20 percent of GDP higher debt would be associated with 40 basis points higher CDS (credit default swap) spreads.

<sup>12</sup> A slightly different way of calculating the benefit (in terms of reducing the expected cost of crises) of lower debt is to consider the probability that at least one crisis occurs over the planning horizon (say, 20 years). At debt of 120 percent of GDP, this probability is 41 percent ( $= 100 \times (1 - (1 - 0.026)^{20})$ ), whereas at 100 percent of GDP, the corresponding probability is 38.5 percent; again, therefore, the estimated benefit is small (0.37 percent of GDP ( $= 100 \times ((0.41 - 0.38) \times 0.15)$ )).

(by deliberately running overall surpluses) is unlikely to exceed the cost of the necessary distortionary taxation.

In such cases, debt-to-GDP ratios should be reduced organically, through output growth, or opportunistically, when less distortionary sources of revenue such as privatization receipts or royalties are available. Within the confines of the formal model, there are two such sources of short-run inelastic revenue: income from initial holdings of public debt and income from the initial stock of private capital. Both of these sources will be inelastic in the short run, but highly elastic in the longer run (because agents can choose to save less in the form of government bonds or private capital when they expect the income to be taxed heavily).

Since there is no explicit tax on the income from government bonds here, the “taxation” takes the form of very low—perhaps negative—real interest rates in the initial period. In a monetary model with sticky prices, this could be achieved by cutting nominal interest rates—as indeed has been the case in advanced economies since the global financial crisis where policy rates have been at historic lows and have even flirted with the zero-bound. Within the confines of the “real” model presented here, low real interest rates must be engineered by cutting taxes on the more elastic tax base (labor), giving a fillip to output and consumption in the initial period, and thus reducing the rate of return payable on the inherited debt.<sup>13</sup>

A similar phenomenon occurs when there is the possibility of taxing capital income. In the initial periods, when the capital stock is fixed (that is, before it depreciates), capital income is inelastic and its taxation largely non-distortionary. Accordingly, the optimal fiscal program calls for heavy taxation to reduce public debt. Very quickly, however, the private capital stock becomes highly elastic (the anticipation of taxation will lead to lower investment) and the optimal tax rate on capital falls to zero.<sup>14</sup> In an economy with an initially inelastic source of revenue, the government should opportunistically pay down some of the inherited debt; the greater the inherited debt, the larger the amount paid down through such taxation.

## STYLIZED FACTS

Since the analytical framework used here is largely normative, it cannot be formally tested against statistical evidence. Nevertheless, it may be interesting to examine the extent to which countries have been following the framework’s prescriptions, and whether some its implications hold empirically. This section summarizes some stylized facts regarding debt, private and public investment, and output growth using two bodies of data: the short span since the 2008 global

<sup>13</sup> See Lucas and Stokey (1983), Debortoli and Nunes (2010, 2012), and Krusell, Martin, and Rios-Rull (2008).

<sup>14</sup> As argued by Chamley (1986), models of optimal taxation generally imply zero taxation of private capital in the long run because private capital becomes a fully elastic source of revenue.

financial crisis, and a longer span covering the period 1960–2008. The advantage of the postcrisis data is that, during the crisis, several advanced economies experienced large increases in public debt that were almost entirely unrelated to public investment. The downside, of course, is that much else was happening in these economies, and it may be too early to tell what will be the medium- to long-run impact of the increase in public debt on growth.

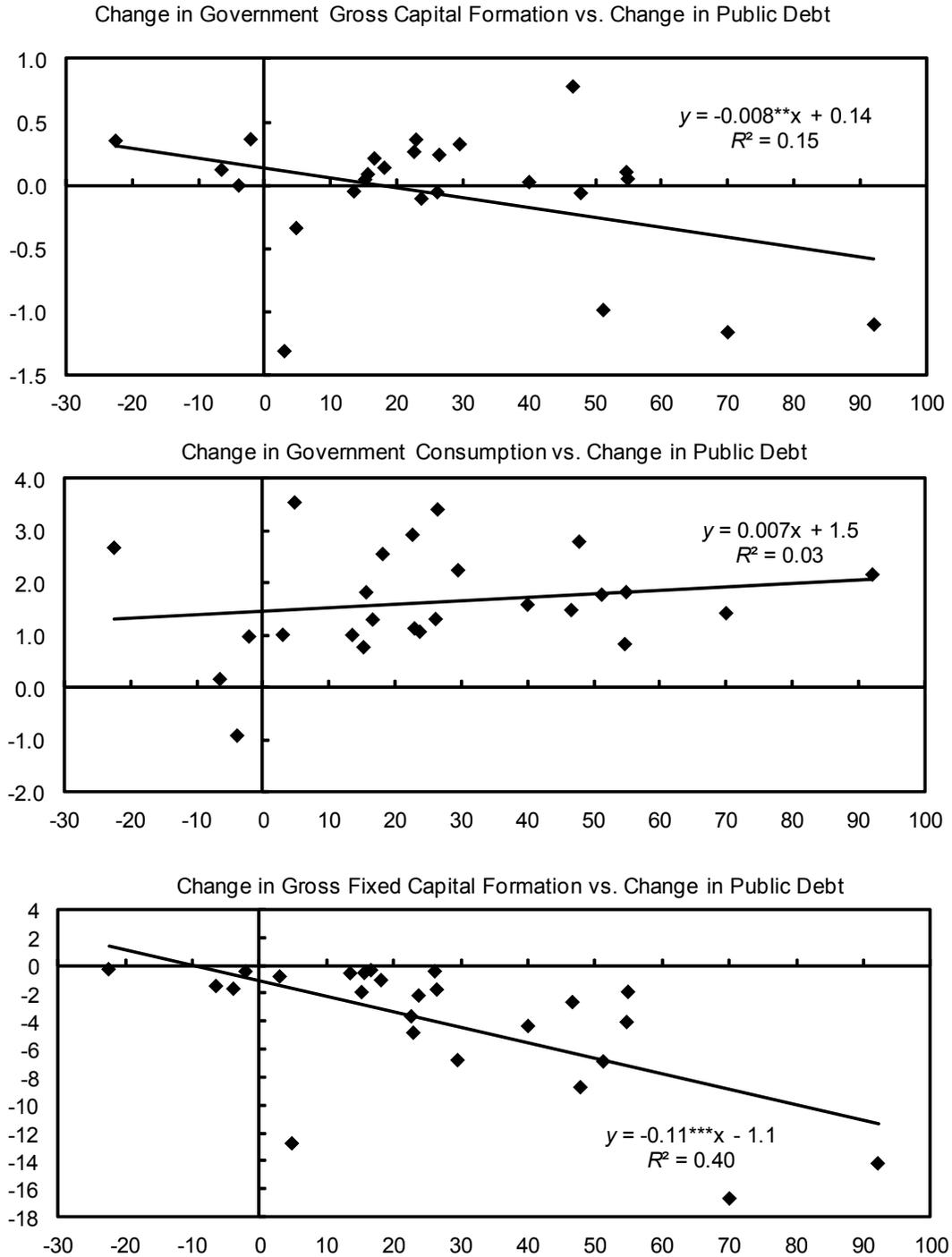
With this caveat in mind, Figure 4 (top panel) reports the correlation between the change in debt over the period 2009–11 and the change in public gross capital formation in percent of GDP (measured as the average for the period 2009–11 relative to average 2005–07). In general, countries that experienced larger increases in public debt were also those that reduced public investment the most, with the association statistically significant at 5 percent level.<sup>15</sup> Of course, sovereigns that experienced the largest increases probably had to do the most belt-tightening as well, so the negative correlation between rising debt and shrinking public investment is not much of a surprise. But it is unlikely to be only the government’s budget constraint at play: public consumption shows no such negative relationship—in fact, the relationship is marginally positive, albeit statistically insignificant (Figure 4, middle panel). This finding likely reflects a pragmatic approach by governments who find it politically easier to cut investment than current expenditure, especially during an economic downturn. But it is noteworthy that the analytical framework developed here would carry the same implication as a normative statement: given the higher inherited debt, and the higher distortionary taxation required to service it, the return to—and therefore the amount of—public investment will be lower. Moreover, again consistent with the analytical framework, the negative relationship between the increase in public debt and the decrease in economywide gross fixed-capital formation holds even more strongly in the data (Figure 4, bottom panel).

Turning to more systematic evidence, Table 1 reports regressions of public investment on current or (five-year) lagged public debt using five-year, non-overlapping averages of these variables for a sample of advanced economies over the period 1960–2008. Again, there is a strong negative relationship between public debt and either current or subsequent public investment (Table 1, columns 1–2). But no such negative relationship can be seen between debt and public consumption (Table 1, columns 3–4).

Beyond the observation that an economy with higher public debt should—and will—do less public investment, the analytical framework implies that, in general, it is better (for both growth and welfare) to live with high debt than to try to reduce it through distortionary taxation. Table 2 reports standard growth regressions (again using 5-year non-overlapping averages for Organisation for Economic Co-operation and Development [OECD] countries). Higher tax rates (measured as revenues in percent of GDP) are indeed associated with lower growth, controlling

<sup>15</sup> We are not claiming that the mechanism emphasized by our model is the only factor at play here: political constraints, for example, would likely have played a role in determining the cutbacks in public investment.

**Figure 4. Advanced Economies: Postcrisis Debt, Public and Private Investment, and Public Consumption**



Sources: IMF and Organization for Economic Co-operation and Development.  
 Note: Change in public debt (in percent of GDP) 2007–11 vs. change in Government Fixed Capital Formation, Government Consumption, Economywide Fixed Capital Formation avg(2005–07)–avg(2009–11).

**Table 1. OECD Countries: Public Debt, and Public Investment and Consumption**

VARIABLES	(1)	(2)	(3)	(4)
	Public Investment	Public Investment	Public Consumption	Public Consumption
Debt/GDP	-1.896*** [-3.954]		6.075*** [5.196]	
Debt/GDP (t-1)		-2.264*** [-3.495]		4.251*** [4.520]
Constant	4.808*** [20.83]	4.878*** [16.48]	14.83*** [27.90]	16.01*** [38.13]
Observations	229	211	274	247
R-squared	0.168	0.211	0.277	0.174
Number of countries	26	26	34	34

Note: Robust t-statistics in brackets.

\*\*\* p<0.01, \*\* p<0.05, \* p<0.1.

**Table 2. OECD Countries: Government Revenues and Real GDP Growth 1/**

VARIABLES	(1)	(2)	(3)	(4)	(5)	(6)
	FE	FE excl. inv	FE incl. debt	A-Bond 2-step collapsed	IV instruments: VAT and SSC rates	FE incl. debt with VAT
Gvt revenues (percent of GDP)	-0.0396** [-2.106]	-0.0424** [-2.349]	-0.0399* [-1.756]	-0.0866** [-2.207]	-0.423** [-2.306]	
Debt/GDP (t-1)			-0.00463 [-0.718]	-0.00559 [-0.341]	0.0164 [0.984]	0.0175 [1.463]
Population growth	0.424 [1.587]	0.532* [1.937]	0.536* [1.892]	0.557* [1.762]	1.424** [1.974]	1.083* [1.923]
Initial GDP	-0.0386*** [-5.450]	-0.0266*** [-3.744]	-0.0219*** [-3.014]	0.00238 [0.556]	-0.0848* [-1.827]	-0.0698** [-2.178]
Terms of trade growth	0.0437 [1.475]	0.0459 [1.411]	0.0383 [1.166]	0.0682 [1.014]	-0.118 [-0.962]	0.154 [1.646]
Terms of trade volatility	2.22e-05 [0.0868]	-3.59e-05 [-0.128]	2.88e-05 [0.105]	-0.000415 [-0.860]	0.00357 [1.412]	-0.00103 [-0.918]
Log(inflation)	-0.00728*** [-3.214]	-0.00280 [-1.099]	-0.00419 [-1.611]	-0.00902 [-1.674]	-0.0249*** [-2.679]	-0.0123*** [-3.340]
Investment (total, percent of GDP)	0.159*** [4.388]			0.271** [2.519]		
VAT rate						-0.00657*** [-4.289]
Constant	0.129*** [6.902]	0.127*** [6.508]	0.128*** [4.643]	0.00906 [0.284]	0.890** [2.186]	0.627** [2.261]
Observations	233	240	227	222	70	107
R-squared	0.707	0.669	0.651		0.596	0.736
Number of countries				27		
Number of instruments				24		
Hansen test p-value				0.235		
A-B AR(1) test p-value				0.000840		
A-B AR(2) test p-value				0.829		

Note: FE= fixed effects; SSC= social security contribution; VAT= value added tax

1/ Dependent variable: Non-overlapping, five-year average real GDP growth, 1960–2008.

Robust t-statistics in brackets.

\*\*\* p<0.01, \*\* p<0.05, \* p<0.1.

Coefficients for time and country dummy variables not shown.

for the other usual growth determinants such as population growth, initial per capita income, terms of trade shocks, inflation, and investment (Table 2, column 1).<sup>16</sup> Part of this effect is through lower investment (again, as the framework would suggest); dropping investment, the negative coefficient on taxes becomes larger in absolute value (Table 2, column 2). Nor are these results likely to be driven by reverse causality: if tax revenues are responding to GDP growth, then the estimated coefficient would be positive; the finding of a statistically significant negative coefficient must therefore be in spite of—rather than because of—any possible endogeneity. Moreover, using the Arellano-Bond estimator or instrumenting using value-added tax (VAT) or social security tax rates yields similar results (Table 2, columns 4–6).

Finally, it is noteworthy that when current or lagged debt is included in the regression (in addition to taxes), the coefficient is negative but insignificant; inherited public debt may be bad for growth (since taxes will need to be somewhat higher to service the debt even if the government choose to live with the higher debt), but what really kills growth is distortionary taxation to service the debt—and, a fortiori, to pay it down.

## CONCLUSIONS

Advanced economies are facing some of the highest ratios of public debt since World War II. For those countries that are not at imminent risk of losing market access, current policy debates center on the appropriate pace at which to pay down public debt. Those who believe that debt is bad for growth favor a rapid reduction in indebtedness, whereas those who stress Keynesian demand management considerations argue for a measured pace of consolidation, perhaps with a ramping up of public investment while interest rates remain at historic lows. Somewhat lost in this debate is the possibility of simply living with (relatively) high debt, and allowing debt ratios to decline organically through output growth. The purpose of this paper is not to provide direct policy advice, but rather to elicit a debate on whether governments should actively seek to pay down public debt or simply live with high debt.

The analytical framework proposed here suggests that this third course, of living with high debt, merits consideration in countries where debt sustainability concerns are not pressing. Indeed, while debt may be bad for growth, it does not follow that it should be paid down as quickly as possible, even abstracting from Keynesian effects on activity and output. At the same time, higher debt may imply lower returns to public capital than prior to the crisis, implying that a scaling-back of investment plans would be appropriate.

<sup>16</sup> Our results are very similar when estimating a cross-section regression of growth over the period 1978–2007. In such regressions, contemporaneous tax revenues are significant, whereas lagged tax revenues and lagged debt are not significant variables. However, the literature on the effect of taxes on growth has been less conclusive, as underlined by Easterly and Rebelo (1993). Mendoza, Milesi-Ferretti, and Asea (1997) also found that although taxes affect investment, the effect on growth is not robust.

The main insights may be summarized in three guiding principles for thinking about public debt and investment dynamics:

- ***Inherited public debt represents a deadweight burden on the economy, reducing both its investment potential and its growth prospects.*** Although the debt may have been incurred for good reasons, the higher the inherited stock, the poorer the economy (by the present value of the distortionary costs of the taxation need to service the debt). Efficiency dictates that the higher inherited debt and corresponding taxation, the lower should be public and private investment, and the slower will be output growth.
- ***When fiscal space is ample—which can never be established through some mechanical procedure—there is a case for simply living with the debt, paying it down only “opportunistically” when non-distortionary sources of revenue are available and letting the debt ratio decline through growth.*** The deadweight loss associated with inherited public debt represents a sunk cost—so, abstracting from rollover risk, there is little purpose in paying it down by raising taxes or cutting otherwise-productive government expenditure. Living with the debt in such circumstances represents the best cost-benefit tradeoff.
- ***Debt should be used to smooth the taxes necessary to finance lumpy government expenditures.*** For public investment, this implies debt-financing of projects whose social marginal product earns at least the market interest rate, but recognizing that the additional debt incurred will need to be serviced with distortionary taxation of factors that may be complementary to public capital, thereby reducing the return to public investment. This cumulative effect on the stock of public capital is likely to be appreciable when public debt is high, implying high level of distortionary taxation.

These are just some guiding principles whose application in any particular case will need to take account of country-specific circumstances. Countries facing imminent risk of a curtailment of market access, or that need to re-establish fiscal space against the risk of contingent liabilities or other shocks, naturally do not have the luxury of living with high debt. For others, the appropriate pace depends on the availability of non- (or less) distortionary sources of tax revenue. And for those countries in the fortunate position of enjoying asset price booms, the message must be that they should seize the opportunity to pay down public debt. In sum, the appropriate response to high levels of public debt depends very much on the extent of available fiscal space and other factors. There is no one-size-fits-all message: be it to pay down the debt to reduce the risk of a funding crisis or to live with the debt, letting the debt ratio decline organically through growth. Countries in the yellow and red zones in terms of fiscal space will not be in a position to “live with the debt.” But nor is it the case that countries with ample space—those firmly in the green zone—should rush to pay down their debt.

### Box 1. A Formal Model of Debt, Investment, and Growth

The economy is closed, and is populated by a single infinitely lived agent who maximizes:

$$\text{Max}_{c_t, l_t, b_t, k_t^p} \sum_{t=0}^{\infty} \beta^t u(c_t, l_t, g_t) \quad (1.1)$$

$$\text{s.t.} \quad c_t + k_t^p + b_t = (1 - \tau_t^L) w_t l_t + R_t k_{t-1}^p + Y_t b_{t-1} \quad (1.2)$$

in which  $c, g, (1-l)$  are consumption of the private and public goods, and of leisure, respectively;  $w$  is the wage rate, taxed at  $\tau^L$ ,  $R$  and  $Y$  are the returns on private capital,  $k^p$ , and bonds,  $b$ . Letting  $\mu_t$  denote the Lagrange multiplier on the budget constraint, the first-order conditions are:

$$\beta^t u_{c,t} = \mu_t \quad (1.3)$$

$$-\frac{u_{l,t}}{u_{c,t}} = (1 - \tau_t) w_t \quad (1.4)$$

$$R_t = Y_t = \frac{u_{c,t-1}}{\beta u_{c,t}} \quad (1.5)$$

The conditions for optimality are intuitive: (1.4) implies that the agent will supply labor to the point that the marginal rate of substitution between consumption and leisure equals the after-tax wage rate; condition (1.5) equates the intertemporal rate of substitution to the return on the government bond, which, by arbitrage, must also equal the return on private capital.

Finally, multiplying (1.2) by the Lagrange multiplier,  $\mu_t$ , summing for all  $t \geq 0$ , using the transversality conditions ( $\lim_{t \rightarrow \infty} \mu_t b_t = \mu_t k_t^p = 0$ ), and exploiting (1.3)–(1.5) provides a convenient summary of the private sector's behavior:

$$\sum_{t=0}^{\infty} \beta^t (u_{c,t} c_t + u_{l,t} l_t) = u_{c,0} (Y_0 b_{-1} + R_0 k_{-1}^p) \quad (1.6)$$

in which  $b_{-1}$  and  $k_{-1}^p$  are the initially inherited stocks of government bonds and private capital. In turn, (1.6) constitutes a "feasibility constraint" that the government must take into account in choosing its optimal fiscal policy.

The simulations shown in the figures assume a Cobb–Douglas production function

$$y_t = k_{t-1}^{g(\theta)} k_{t-1}^{p(\alpha)} l_t^{(1-\alpha)}, \quad (1.7)$$

in which  $\alpha = 0.33$  and  $\theta = 0.1$  (these, and other parameters, are standard from the literature).

The utility function is iso-elastic:

$$u(c, l, g) = (1 - \phi_g) \left( \phi_c \frac{c^{1-\sigma_c}}{1-\sigma_c} + (1 - \phi_c) \frac{(1-l)^{1-\sigma_l}}{1-\sigma_l} \right) + \phi_g \frac{g^{1-\sigma_g}}{1-\sigma_g} \quad (1.8)$$

in which  $\phi_c = 0.3$ ;  $\phi_g = 0.1$ ;  $\sigma_c = 1$ ;  $\sigma_l = 1$ ;  $\sigma_g = 1$  (this calibration implies utility is logarithmic). Other parameters are the discount rate,  $\beta = 0.96$ , and the rate of depreciation,  $\delta = 0.1$ .

### Box 2. Optimal Fiscal Policy

The government commits to a fiscal plan  $\{\tau_t, g_t, b_t, k_t^g\}_{t=0}^{\infty}$  (in which  $k_t^g$  is public capital) to maximize the agent's utility, subject to the economywide resource constraint and the "feasibility constraint" that summarizes private sector behavior (Box 1 (1.6)). Formally:

$$\text{Max}_{\tau_t, g_t, b_t, k_t^g} \sum_{t=0}^{\infty} \beta^t u(c_t, l_t, g_t) \quad (2.1)$$

$$\text{s.t. } c_t + g_t + k_t^p - (1 - \delta)k_{t-1}^p + k_t^g - (1 - \delta)k_{t-1}^g = F(k_{t-1}^g, k_{t-1}^p, l_t) \quad (2.2)$$

$$\sum_{t=0}^{\infty} \beta^t (u_{c,t} c_t + u_{l,t} l_t) = u_{c,0} (Y_0 b_{-1} + R_0 k_{-1}) \quad (2.3)$$

The optimal fiscal program is:

$$F_{k_t^p} = F_{k_t^g}, u_{g_t} = \mu_t \forall t \geq 0 \quad (2.4)$$

$$-\{u_{l,t} + \lambda(u_{ll,t} l_t + u_{l,t})\} = \{u_{c,t} + \lambda(u_{cc,t} c_t + u_{c,t})\} F_{l,t} \forall t \geq 1 \quad (2.5)$$

$$u_{c,t} + \lambda(u_{cc,t} c_t + u_{c,t}) = \beta(u_{c,t+1} + \lambda(u_{cc,t+1} c_{t+1} + u_{c,t+1})) (F_{k_t^g} + (1 - \delta)) \forall t \geq 1 \quad (2.6)$$

in which  $\mu_t, \lambda$  are the Lagrange multipliers on resource and feasibility constraints respectively.

When taxes are lump-sum, and thus non-distortionary, the feasibility constraint does not bind, so  $\lambda = 0$ , in which case (2.5)–(2.6) reduce to:

$$-u_{l,t} / u_{c,t} = F_{l,t} \forall t \geq 1 \quad (2.7)$$

$$(u_{c,t} / \beta u_{c,t+1}) = (F_{k_t^p} + (1 - \delta)) = (F_{k_t^g} + (1 - \delta)) \forall t \geq 1 \quad (2.8)$$

But these are just the analogs to the representative agent's first-order conditions, (1.4)–(1.5). Condition (2.7) equates the marginal rate of substitution between consumption and leisure to the marginal product of labor, implying from (1.4), that the wage should be the marginal product of labor. Condition (2.8) states that private and public investment should be undertaken to the point that the respective marginal products equal the intertemporal rate of substitution, which, from (1.5), is the market interest rate.

When only distortionary taxes are available, however,  $\lambda > 0$ , and the impact on the government's ability to raise revenues needs to be taken into account. Therefore, social rather than private marginal products and rates of substitution must be used, as in (2.4)–(2.6). For example, investment may yield a higher social return than the private return because, by raising the marginal product of labor, it makes it easier to raise labor taxes. Nevertheless, a key result is that, even with  $\lambda > 0$ , the implicit discount rate used by the government in its optimal fiscal program equals the market interest rate. This holds exactly in the case of iso-elastic utility, and approximately otherwise. Substituting (1.8) into (2.6), yields:

$$(u_{c,t} / \beta u_{c,t+1}) = R_t = F_{k_t^p} + (1 - \delta) = F_{k_t^g} + (1 - \delta) \quad (2.9)$$

Condition (2.9) states that even though taxes are distortionary, *the market interest rate equals the representative agent's discount factor*—which is also the discount factor used by the government in setting optimal policy (for example, when deciding whether to repay debt or defer taxation).

Finally, in period 0, the first-order conditions (2.5)–(2.6) are:

$$-\{u_{l,0} + \lambda(u_{ll,0} l_0 + u_{l,0})\} = \{u_{c,0} + \lambda(u_{cc,0} c_0 + u_{c,0} - u_{cc,0} (Y_0 b_{-1} + R_0 k_{-1}^p))\} F_{l,0} \quad (2.10)$$

$$u_{c,0} + \lambda(u_{cc,0} c_0 + u_{c,0} - u_{cc,0} (Y_0 b_{-1} + R_0 k_{-1}^p)) = \beta(u_{c,1} + \lambda(u_{cc,1} c_1 + u_{c,1})) (F_{k_1^g} + (1 - \delta)) \quad (2.11)$$

These differ from their period  $t \geq 1$  analogs by the term  $u_{cc,0} (Y_0 b_{-1} + R_0 k_{-1}^p)$ , which pertains to the optimality, as discussed in the text, of opportunistically taxing inelastic tax bases—income from the initial stocks of government bonds and of private capital.

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